

Regge Exchange Contribution to Deeply Virtual Compton Scattering

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Recently we have shown that exclusive QCD photon-induced reactions at low Mandelstam- t are best described by Regge exchanges in the entire scaling region, and not only for low values of Bjorken- x . In this paper we explore this crucial Regge behavior in Deeply Virtual Compton Scattering from the point of view of collinear factorization, with the proton tensor written in terms of Generalized Parton Distributions, and we reproduce this feature. Thus it appears that in the Bjorken limit, a large class of hard, low- t exclusive processes are more sensitive to the meson cloud of the proton than to its fundamental quark structure. These process will then be described most efficiently by process-dependent Regge Exclusive Amplitudes rather than by universal Generalized Parton Distributions. We introduce such Regge Exclusive Amplitudes for Deeply Virtual Compton Scattering.

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I. INTRODUCTION

Nowadays it is common to consider deeply virtual exclusive electroproduction of mesons or photons in the context of generalized parton distributions (GPD's) [1, 2, 3, 4, 5]. In the case of deeply virtual Compton scattering (DVCS) [6, 7], a single initial quark near its mass shell becomes highly virtual after it interacts with the off-shell photon. This virtual quark propagates essentially without interaction with the quark and gluon spectators until it radiates a real photon. To produce a final hadron in place of a real photon, the off-shell quark can radiate a hard gluon that enhances its correlation with soft quarks and antiquarks around the target, and hence increases the probability of hadronization into a single meson.

In the language of QCD factorization [8, 9], exchange of a single off-shell quark between the interaction point for the virtual photon and real photon (or meson, in the case of hard exclusive meson production) corresponds to application of the operator product expansion to a product of electromagnetic currents and/or meson interpolating fields. In the center of mass frame between the initial quark and virtual photon, the incoming quark has one large momentum component and is nearly on its mass shell. To obtain amplitudes in terms of generalized parton distributions it is necessary to ignore the other, small momentum components and quark masses.

An alternative approach to exclusive processes involving strong interactions is based on identifying singularities of the scattering amplitude close to the physical region of the relevant kinematical variables. In particular one expects physical states with quantum numbers of the t -channel to dominate processes involving two-

to-two particle scattering, $ab \rightarrow cd$ at low momentum transfer to the target, $t = (p_d - p_b)^2 < 0$ and large values of s (the square of the center of mass energy) $s = (p_a + p_b)^2 \gg -t$. Inclusion of all allowed t -channel exchanges leads to the Regge-type dependence, of the scattering amplitude $A(s, t)$

$$A(s, t) \sim s^{\alpha(t)} \quad (1)$$

on the center of mass energy s where for low momentum transfer $-t \lesssim 1 \text{ GeV}^2$. In Eq. (1) the intercept $\alpha(t)$ of the Regge trajectory is a positive number less than one (with exception of diffractive scattering which in this language corresponds to the Pomeron exchange). The microscopic picture of exclusive electroproduction suggested by Regge phenomenology differs substantially from an explanation predicated on generalized parton distributions. Qualitatively, the Regge picture corresponds to virtual photon scattering from quarks in the meson cloud around the bare nucleon, as opposed to the virtual photon scattering from quarks in the core of the nucleon, as implied by the connection between GPD's and the spatial distribution of valence quarks [10, 11].

A distinguishing feature of the GPD mechanism is amplitude scaling in terms of Bjorken variables, *i.e.* at fixed momentum transfer and mass of the produced hadron (or photon) the hadronic part of electroproduction amplitudes [12] is predicted to be a function of $-q^2 = Q^2$, the photon virtuality and $x_B \equiv Q^2/(2\nu)$ where $\nu = p_a \cdot p_b$ is the energy of the virtual photon in the target rest frame. Furthermore at fixed Bjorken x_B QCD makes specific predictions for the leading order large- Q^2 dependence. In the Regge case, the amplitude is expected to be a function of both Q^2 and ν .

Recent results on exclusive vector and pseudoscalar

meson production from JLab [13] and HERMES [14] generally do not exhibit the Q^2 scaling predicted by QCD. In the case of meson production, the $\gamma^*p \rightarrow Mp$ cross-section is predicted to fall as $1/(Q^2)^n$ with $n = 3$, while JLab ω production data and HERMES π^+ data taken in a similar kinematic range give $n \sim 2$. Earlier data [15] on ρ_0 production might be consistent with QCD expectations, but these results appear to be softer than the $n = 3$ predicted by QCD scaling.

DVCS data from Hall A at Jefferson Laboratory [16] and HERMES [14] appear to be consistent with the Q^2 -independent amplitude predicted by QCD [17], however the available Q^2 window is quite small, from 1.5 – 2.5 GeV² and within the published experimental errors one cannot rule out a power-like dependence of the amplitude, $A \propto (Q^2)^\alpha$, with α as large as 0.25. Perhaps even more surprising, "standard" Regge-exchange models have proved successful in describing a variety of differential cross sections [13, 18], in the kinematical range where scaling would be expected based on comparisons with deep inelastic scattering (DIS). As we see it, a fundamental question is whether the success of the Regge picture is accidental. If not, this immediately raises the question of how one can disentangle scattering off the meson cloud from effects of nucleon tomography.

It is well known that Regge exchanges also contribute to DIS structure functions [19], but their contribution is restricted to very low $x_B \sim 0$. Once it was realized that Regge exchange may play a significant role in exclusive electroproduction, attempts have been made to incorporate Regge effects using analogies with DIS, *i.e.* to restrict Regge contributions in exclusive electroproduction reactions to low- x_B so that scaling is not otherwise modified [20, 21, 22, 23]. To the best of our knowledge it has not been proven that Regge contributions should only contribute to exclusive amplitudes in this domain, and in fact in Ref. [24] we provide arguments that, in a certain kinematic regime, Regge effects should be substantial even at large x_B .

In this paper, we investigate further this issue. In Ref. [24] we analyzed hard exclusive reactions by examining the high-energy behavior of t -channel exchange processes. Here we will show that utilization of an s -channel framework, in which one analyzes the "hand-bag" diagrams that are used in extracting GPDs, leads to the same conclusions reached in Ref. [24], *i.e.* we show that in the region of high energy and small t , Regge effects should make sizeable contributions to hard exclusive amplitudes.

In this s -channel formalism we are able to explore the interplay between Regge behavior in the parton-nucleon amplitude and the hard interaction induced by the virtual photon. We will show that there are crucial differences between DIS and DVCS hand-bag diagrams which make Regge components of the soft parton-nucleon amplitude much more pronounced for DVCS than for DIS. We find that the difference between these processes arises when one attempts a collinear factorization of the quark

propagators occurring in these processes. In the presence of Regge behavior in the parton-nucleon amplitude, the DVCS formalism is ill-defined. We then compute the hand-bag diagram using the full hard quark propagator. For hard exclusive processes, the divergent terms that are introduced produce a non-analytic, non-scaling dependence on the photon virtuality.

This has the following effect on the hard exclusive amplitudes. First, the breakdown of factorization means that the soft amplitudes are not universal, but are process-dependent. Second, in the region of small t Regge effects will make substantial contributions to DVCS and exclusive meson production. Third, the Q^2 behavior of these hard exclusive processes should be different from that predicted from scaling arguments.

Our paper is organized as follows. In the following Section we introduce the framework and consider the case of collinear factorization. We review both DIS and DVCS reactions, and we show that the DVCS formalism is ill-defined in the presence of Regge-like behavior in the parton-nucleon amplitude. In Section III we compute the hand-bag diagram with the full hard quark propagator and show how the divergent would-be collinear factorization forces a non-analytical, non-scaling dependence on the photon virtuality. We derive the Q^2 behavior for hard exclusive processes and show how it differs from scaling predictions, and how this Q^2 behavior is related to the leading Regge trajectories. We analyze existing DVCS and exclusive meson data, and show that their Q^2 behavior is, at least qualitatively, consistent with our predictions.

II. COLLINEAR FACTORIZATION IN PRESENCE OF REGGE ASYMPTOTICS

The hadronic tensor that describes electromagnetic transitions in double diagonal DIS $\gamma^*p \rightarrow \gamma^*p$ or off-diagonal $\gamma^*p \rightarrow \gamma p$ DVCS reactions, is given by

$$T^{\mu\nu} = i \int d^4z e^{i\frac{q'+q}{2}z} \langle p' \lambda' | T J^\mu(z/2) J^\nu(-z/2) | p \lambda \rangle. \quad (2)$$

In Eq. (2), q is the four momentum of the virtual photon, $q^2 < 0$, $q' = q + p - p' \equiv q - \Delta$, and $q'^2 = 0$ is the momentum of the real photon produced in DVCS. In the case of DIS, $q'^2 = q^2$ and $\Delta = 0$ and the DIS cross section is proportional to the discontinuity of T across the cut in $(p + q)^2$. Even though we will explicitly consider only the kinematics relevant for either DIS or DVCS the analysis can easily be extended to the more general case of arbitrary time-like q' which is relevant, for example for meson electroproduction. The currents are given by $J^\mu(z) = \sum_q e_q J_q^\mu(z)$, $J_q^\mu(z) = \bar{\psi}(z) \gamma^\mu \psi(z)$ where ψ is the quark field operator and e_q is the quark charge. Throughout this paper we will consider a single quark flavor. For large Q^2 the z -integral peaks at $z^2 \sim 1/Q^2$ and using the leading order operator product expansion of QCD we replace the product of the two currents by a product of

two quark field operators and a free propagator between

the photon interaction points $(z/2, -z/2)$

$$\begin{aligned}
 T^{\mu\nu} &= -e_q^2 \int \frac{d^4 z d^4 k}{(2\pi)^4} \frac{\left[\gamma^\mu \left(\not{k} + \frac{\not{q} + \not{q}'}{2} \right) \gamma^\nu \right]_{\alpha\beta}}{\left(\frac{q_+ q'_+}{2} + k \right)^2 + i\epsilon} \langle p' \lambda' | T \bar{\psi}_\alpha(z/2) \psi_\beta(-z/2) | p \lambda \rangle [e^{-ikz} - e^{+ikz} (\mu \leftrightarrow \nu)] \\
 &\equiv -ie_q^2 \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{\left[\gamma^\mu \left(\not{k} + \frac{\not{q} + \not{q}'}{2} \right) \gamma^\nu \right]_{\alpha\beta}}{\left(\frac{q_+ q'_+}{2} + k \right)^2 + i\epsilon} - \frac{\left[\gamma^\nu \left(-\not{k} + \frac{\not{q} + \not{q}'}{2} \right) \gamma^\mu \right]_{\alpha\beta}}{\left(\frac{q_+ q'_+}{2} - k \right)^2 + i\epsilon} \right\} A_{\beta\alpha}(k, \Delta, p, \lambda, \lambda').
 \end{aligned} \tag{3}$$

Here A is the parton-nucleon scattering amplitude untruncated, with respect to the parton legs,

$$\begin{aligned}
 A_{\beta\alpha} &\equiv A_{\beta\alpha}(k, \Delta, p, \lambda', \lambda) \\
 A_{\beta\alpha} &= -i \int d^4 z e^{-ikz} \langle p' \lambda' | T \bar{\psi}_\alpha(z/2) \psi_\beta(-z/2) | p \lambda \rangle.
 \end{aligned} \tag{4}$$

As in Refs. [25, 26, 27, 28, 29], we assume that despite its non-physicality, the analytical properties of the parton-nucleon amplitude display structures in the complex plane similar to conventional hadron scattering amplitudes. This is necessary if such amplitudes are to be of any use at all, *i.e.* if they are to be connected to asymptotic properties of QCD [30]. The T -ordered product could then be replaced by a normal ordered product corresponding to generalized parton distributions [32]. For the purpose of our study it will be more efficient to deal directly with the T -ordered amplitudes. The parton-nucleon amplitude is a function of four variables and the nucleon helicities. The variables are $k_1^2 = (\Delta/2 - k)^2$, $k_2^2 = (-\Delta/2 - k)^2$ the (virtual) masses of the incoming and outgoing partons ($\Delta = p' - p = q - q'$), $s = (p + k_1)^2 = [(p + p')/2 - k]^2$ is the square of the center of mass energy in the s -channel, $u = (p' - k_1)^2 = [(p' + p)/2 + k]^2$ is the square of the center of mass energy in the u -channel. Together with the four-momentum transfer, $t = (p' - p)^2 = \Delta^2$ they satisfy $s + t + u = k_1^2 + k_2^2 + 2M^2$ where M is the nucleon mass.

To obtain DIS scaling relations it is necessary to assume that the parton-nucleon amplitude has cuts for positive s and u . We will be interested primarily in the implications of the high- s or u behavior at low t where the amplitude is expected to be helicity conserving. Further-

more to reproduce the scaling limit of DIS and to preserve current conservation, the dependence on the parton spin (Dirac) indices must be of the form $A_{\beta\alpha} \propto \not{k}_{\beta\alpha}$, or $[\gamma_5 \not{k}]_{\beta\alpha}$. The former (latter) contributes respectively to the symmetric (antisymmetric) parts of the hadronic tensor $T^{\mu\nu}$.

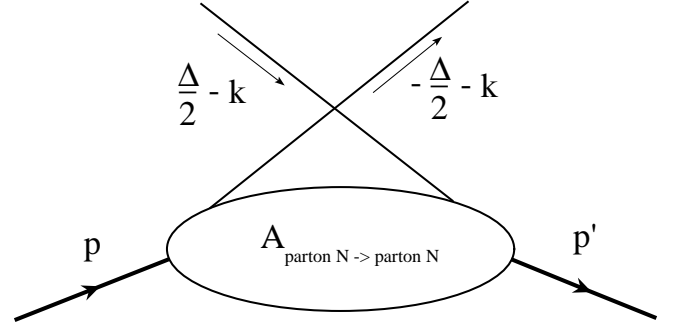


FIG. 1:
Parton-nucleon scattering amplitude

Finally for fixed- t we arrive at the general representation of the parton-nucleon amplitude in the form,

$$A = A^+ \frac{\not{k}_{\beta\alpha}}{4} \delta_{\lambda'\lambda} + A^- \frac{[\gamma_5 \not{k}]_{\beta\alpha}}{4} \tau_{\lambda'\lambda}^3, \tag{5}$$

with the factor of $1/4$ introduced for later convenience and with the amplitudes A^\pm having the Mandelstam representation,

$$A^\pm = (2\pi)^4 \int \frac{d\mu_1^2 d\mu_2^2 dm^2}{(\mu_1^2 - k_1^2 - i\epsilon)(\mu_2^2 - k_2^2 - i\epsilon)} \left[\frac{\rho_s^\pm(\mu_1^2, \mu_2^2, m^2, t)}{m^2 - s - i\epsilon} \pm \frac{\rho_u^\pm(\mu_1^2, \mu_2^2, m^2, t)}{m^2 - u - i\epsilon} \right] + \text{subtractions}. \tag{6}$$

At asymptotically high energies, the quark and antiquark structure functions are becoming identical which implies that the s and u -channel spectral functions become identical, and so for large m^2 $\rho_u^\pm \sim \rho_s^\pm$. These amplitudes are in principle different in the valence (finite m^2) region. Even though we are primarily interested in the large- m^2 region we will distinguish between the s and u spectral functions in order to be able to keep track of quark and antiquark contributions. The dependence of the spectral density ρ^\pm on μ_1^2 and μ_2^2 determines the dependence of the parton-nucleon scattering amplitude on parton virtualities. In perturbation theory [25] one

would have $\rho \propto \delta(\mu_1^2 - m_q^2)\delta(\mu_2^2 - m_q^2)$ where m_q is the bare quark mass. For the bound state nucleon, however, ρ is expected to be softer [26, 29], *e.g.* in order to reproduce correctly the large Q^2 fall-off of the form factors [33]). The spin structure of A could be more complicated than given by the two terms in Eq. (5), for example there could be terms proportional to \not{p} , \not{p}' , or $\not{p}\gamma_5$ *etc.* [26]. As will be clear from the discussion that follows, however, it is the terms proportional to \not{k} that lead to the Regge behavior of the structure functions and thus will be considered here. Without loss of generality we can take

$$\rho_{u,s}^\pm(\mu_1^2, \mu_2^2, m^2) \rightarrow \rho_{u,s}^\pm(m^2, t)(\mu^2)^n \frac{d^n}{d(\mu^2)^n} [\delta(\mu_1^2 - \mu^2)\delta(\mu_2^2 - \mu^2)], \quad (7)$$

with $n \geq 1$, where for simplicity we use a single scale μ for both partons (inclusion of charge symmetry breaking effects is an obvious generalization). The most general spectral density can always be written as a linear combination of functions of this type $\rho = \sum_n c_n \rho_n$. Henceforth we will omit the subindex on ρ_n . As we have already discussed, for low m^2 this amplitude is expected to be sensitive to poles and cuts associated with low energy resonances and few-particle production thresholds. For large m^2 it is expected to be dominated by the leading Regge trajectory,

$$\rho_{u,s}^\pm(m^2, t) = \rho_{u,s,V}^\pm(m^2, t) + \rho_{u,s,R}^\pm(m^2, t). \quad (8)$$

For large m^2 , the valence part $\rho_{u,s,V}^\pm(m^2)$ falls off with m^2 and does not require subtractions, On the contrary for large m^2 the Regge part, $\rho_{u,s,R}^\pm(m^2)$ behaves as

$$\rho_{u,s,R}^\pm(m^2, t) \rightarrow \beta_{u,s}^\pm(t) \left(\frac{m^2}{\mu^2} \right)^{\alpha_{u,s}^\pm(t)}, \quad (9)$$

where $0 < \alpha_{u,s}^\pm(t) < 1$ for small t and requires one subtraction in Eq. 6. Here we consider only the quark contribution, as opposed to gluon exchanges which lead to diffractive, Pomeron-type contributions with $\alpha > 1$. These could also be effectively included but would require additional subtractions. As we are interested in the low- t limit, we have approximated the intercepts and residues by their values in the limit $t \rightarrow 0$.

In the following we will be interested in the role of the Regge (high energy) component and thus the parton-nucleon amplitude can be written,

$$A^\pm(s, u, k_1^2, k_2^2) = (2\pi)^4 \int dm^2 \left\{ \left[\frac{\rho_s^\pm(m^2)}{m^2 - s - i\epsilon} - \frac{\rho_{s,R}^\pm(m^2)}{m^2 - i\epsilon} \right] \pm (s \rightarrow u) \right\} I_n \frac{1}{(\mu^2 - k_1^2 - i\epsilon)(\mu^2 - k_2^2 - i\epsilon)}, \quad (10)$$

where in Eq. (10), $I_n = (\mu^2)^n d^n/d(\mu^2)^n$. It should be noted that as long as s and u channel spectral functions are identical, subtractions are really only necessary for A^+ while they cancel in A^- . We are now in position

to evaluate the two diagrams (direct and crossed) that contribute to the hadronic tensor W as shown in Fig. 2.

For the symmetric part the leading contribution in the Bjorken limit is given by

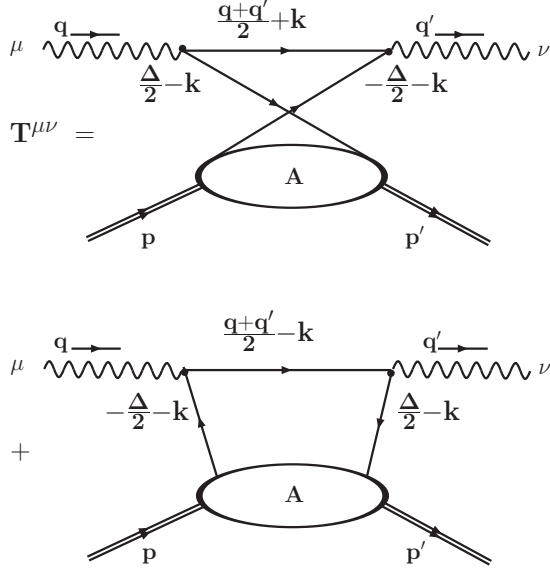


FIG. 2:

u and s channel contributions to the DVCS amplitude

$$\begin{aligned}
 T_+^{\mu\nu} = & -\delta_{\lambda'\lambda} e_q^2 \int \frac{d^4k dm^2}{i} \left[\frac{\rho_s^+(m^2)}{\left(\frac{p+p'}{2} - k\right)^2 - m^2 + i\epsilon} - \frac{\rho_s^+(m^2)}{-m^2 + i\epsilon} + (s \rightarrow u, k \rightarrow -k) \right] \\
 & \times I_n \left[\frac{1}{\left[\left(\frac{\Delta}{2} - k\right)^2 - \mu^2 + i\epsilon\right] \left[\left(\frac{\Delta}{2} + k\right)^2 - \mu^2 + i\epsilon\right]} \right] \\
 & \times \left[\frac{\left(k + \frac{q+q'}{2}\right)^\mu k^\nu + (\mu \leftrightarrow \nu) - g^{\mu\nu} \left(k + \frac{q+q'}{2}\right) \cdot k}{\left(\frac{q+q'}{2} + k\right)^2 + i\epsilon} - \frac{\left(-k + \frac{q+q'}{2}\right)^\mu k^\nu + (\mu \leftrightarrow \nu) - g^{\mu\nu} \left(-k + \frac{q+q'}{2}\right) \cdot k}{\left(\frac{q+q'}{2} - k\right)^2 + i\epsilon} \right] . \quad (11)
 \end{aligned}$$

Similarly the leading contribution to the antisymmetric part can be written,

$$T_-^{\mu\nu} = -\tau_{\lambda'\lambda}^3 e_q^2 \int \frac{d^4 k dm^2}{i} \left[\frac{\rho_s^-(m^2)}{\left(\frac{p+p'}{2} - k\right)^2 - m^2 + i\epsilon} - \frac{\rho_s^-(m^2)}{-m^2 + i\epsilon} - (s \rightarrow u, k \rightarrow -k) \right] \\ \times I_n \left[\frac{-i\epsilon^{\mu\rho\nu\eta}}{\left[\left(\frac{\Delta}{2} - k\right)^2 - \mu^2 + i\epsilon\right] \left[\left(\frac{\Delta}{2} + k\right)^2 - \mu^2 + i\epsilon\right]} \right] \left[\frac{\left(k + \frac{q+q'}{2}\right)_\rho k_\eta}{\left(\frac{q+q'}{2} + k\right)^2 + i\epsilon} + \frac{\left(-k + \frac{q+q'}{2}\right)_\rho k_\eta}{\left(\frac{q+q'}{2} - k\right)^2 + i\epsilon} \right]. \quad (12)$$

To obtain an expression in terms of structure functions or generalized parton amplitudes, one applies a collinear factorization to the quark propagator in the last square bracket in Eqs. (11) and (12). We will first consider the diagonal case, $q = q'$. In this case T is the analog of the hadronic amplitude for forward virtual Compton scattering, whose imaginary part is proportional to the DIS cross-section.

A. The DIS Reaction $\gamma^* p \rightarrow \gamma^* p$

It is convenient to express all momenta in terms of light cone components, $a^\mu = (a^+, a^-, a_\perp)$ with $a^\pm = a^0 \pm a^z$, $a_\perp = (a^1, a^2)$ and to choose a frame in which, $p = p' = (P^+, M^2/P^+, 0_\perp)$, $q = q' = (0, Q^2/x_B P^+, Q_\perp)$, with $-q^2 = -q'^2 = Q^2 = Q_\perp^2$. Since the nucleon mass M does not play a role in our discussion, for simplicity we will set it to zero. The hard quark propagators (the term in the last square bracket in Eqs. (11,12)) become

$$\frac{1}{\left(\frac{q+q'}{2} \pm k\right)^2 + i\epsilon} \rightarrow \frac{x_B}{Q^2} \frac{1}{(-x_B \pm \frac{k^+}{P^+} + i\epsilon)} \quad (13)$$

where following the collinear approximation $k \propto P$ we have ignored terms of the order $|k_\perp|/\sqrt{Q^2}$. The leading contribution to the numerator comes from the terms that maximally involve the photon momentum; the term in the last square bracket in Eq. (11) can be written as

$$[\cdots] = [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \frac{(k^+/P^+)^2}{\left(\frac{k^+}{P^+}\right)^2 - x_B^2 + i\epsilon} \quad (14)$$

where we have introduced the vectors, $n^\mu = (0^+, 2, 0_\perp)$ ($n \cdot a = a^+$) and $\tilde{p}^\mu \equiv p^\mu/P^+$. In the next step we combine all of the propagators using the Feynman parametrization, and we obtain

$$T_+^{\mu\nu} = -\delta_{\lambda'\lambda} e_q^2 [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \int \frac{d^4 k}{i} \int dm^2 \int_0^1 dx \left[\frac{(k^+/P^+)^2}{\left(\frac{k^+}{P^+}\right)^2 - x_B^2 + i\epsilon} \right] \\ \times \left[\rho_s^+(m^2) I_n \left(\frac{2(1-x)}{[(k-xp)^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} - \frac{1}{-m^2(k^2 - \mu^2 + i\epsilon)^2} \right) + (s \rightarrow u, k \rightarrow -k) \right]. \quad (15)$$

Finally we perform the k^- and k_\perp integrals using [34]

$$\int \frac{dk^- d^2 k_\perp}{2i(k^2 + a^2 + i\epsilon)^\alpha} = \pi^2 \frac{\Gamma(\alpha-2)}{\Gamma(\alpha)} \frac{\delta(k^+)}{(a^2 + i\epsilon)^{\alpha-2}} \quad (16)$$

to obtain,

$$T_+^{\mu\nu} = \delta_{\lambda'\lambda} e_q^2 [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \int_0^1 dx \frac{2x}{x_B^2 - x^2 - i\epsilon} [f_q(x) + \bar{f}_q(x)] \quad (17)$$

Here $f_q(x)$, $\bar{f}_q(x)$ are the quark and antiquark structure functions, respectively, which are given by

$$\begin{aligned}
f_q(x) &= \frac{\pi^2}{2} \mu^2 \theta(1-x) \int dm^2 \rho_s^+(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2 + (1-x)\mu^2]^2} = f_V(x) + f_R(x) \\
\bar{f}_q(x) &= \frac{\pi^2}{2} \mu^2 \theta(1-x) \int dm^2 \rho_u^+(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2 + (1-x)\mu^2]^2} = \bar{f}_R(x)
\end{aligned} \tag{18}$$

There is no “valence” contribution to the antiquark distribution. Increasing n produces more powers of $(1-x)$ that soften the propagator, form factors, and simultaneously the $x \rightarrow 1$, end-point behavior of the parton distribution functions (PDFs), as dictated by the Drell-

Yan-West relation [35]. The valence part of the spectral function vanishes in the limit of large- m^2 , which implies that the valence structure functions are proportional to x as $x \rightarrow 0$. The low- x behavior originating from the Regge part of the spectral function is given by

$$f_R(x) = (\mu^2)^{1-\alpha_s^+} \frac{x\pi^2\beta_s^+}{2} I_{n-1} \int_0^\infty \frac{dm^2 (m^2)^{\alpha_s^+}}{(xm^2 + \mu^2)^2} \rightarrow (\mu^2)^{1-\alpha_s^+} I_{n-1} \frac{\pi^2\beta_s^+}{2(\mu^2)^{1-\alpha_s^+}} \left[\frac{\pi\alpha_s^+}{\sin \pi\alpha_s^+} \frac{1}{x^{\alpha_s^+}} \right] \equiv \frac{\gamma_{\alpha_s^+}}{x^{\alpha_s^+}} \tag{19}$$

and for the antiquark distribution $\bar{f}_q(x)$ one needs to replace $s \rightarrow u$. As expected the small- x behavior of the structure function is determined by the leading high-energy behavior of the parton-nucleon amplitude.

A similar analysis for the antisymmetric part, $T_-^{\mu\nu}$, gives

$$T_-^{\mu\nu} = ie_q^2 \epsilon^{\mu\nu}_{03} \tau_{\lambda'\lambda}^3 \int_0^1 \frac{2x_B}{x_B^2 - x^2 - i\epsilon} [\Delta f_q(x) + \Delta \bar{f}_q(x)] , \tag{20}$$

where

$$\begin{aligned}
\Delta f_q(x) &= \frac{\pi^2}{2} \mu^2 \theta(1-x) \int dm^2 \rho_s^-(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2 + (1-x)\mu^2]^2} = \Delta f_V(x) + \Delta f_R(x) \\
\Delta \bar{f}_q(x) &= \frac{\pi^2}{2} \mu^2 \theta(1-x) \int dm^2 \rho_u^-(m^2) I_{n-1} \frac{x(1-x)^2}{[xm^2 + (1-x)\mu^2]^2} = \Delta \bar{f}_R(x).
\end{aligned} \tag{21}$$

Since antiquarks are expected to dominate in the sea region, the valence part $\rho_{u,V}^-$ can be neglected in this region. The low- x behavior of the spin dependent structure functions is determined by the Regge part and is proportional to $1/x^{\alpha_u^-}$ or $1/x^{\alpha_s^-}$ for $\Delta f_q(x)$ or $\Delta \bar{f}_q(x)$, respectively. We note that the subtraction terms do not contribute to the hadronic tensor. This is related to the small- x behavior of the structure functions, which are integrable over the low- x region since we assume $\alpha < 1$. The hard propagators in the collinear approximation do not spoil the convergence of the integrals over low- x . It

is important to realize, however, that this need not be the case in general. For example in the scalar model it was shown that the full $T^{\mu\nu}$ amplitude has a constant component (independent of Q^2 and x_B), the so-called $J = 0$ pole contribution in the language of Regge phenomenology. This component originates from the seagull coupling of both photons to the quark at the same space-time point, as required by QED gauge invariance. This interaction alone leads to a divergent contribution of the form $\int_0 dx f_q(x)/x$ (as opposed to $\int_0 dx f_q(x)$ found above) which gets regulated as $x \rightarrow 0$ precisely by the

subtraction term [27, 28, 29]. Thus in the scalar case the subtraction term is essential for producing a finite Compton amplitude.

From this discussion it should be clear that the convergence of the low- x integration may be a special rather than a general feature of these amplitudes. In Sec. IIC we show that convergence arises for DVCS in a different manner than for DIS.

B. Normalization

The structure functions $f_q(x)$ and $\bar{f}_q(x)$ represent probability densities for finding a quark or antiquark of a particular flavor q in the nucleon and as such need to be normalized to the net number of quarks of that flavor in the proton, (*e.g.* $n_q = (0, 1, 2)$ for s, d and u quarks in the proton, respectively)

$$\int_0^1 dx [f_q(x) - \bar{f}_q(x)] = n_q. \quad (22)$$

Below we verify that this is consistent with the normalization of the vector current which is also sensitive to quark densities. The normalization of the diagonal matrix element of the electromagnetic current, $J^+(0) = e_q \bar{\psi}(0) \gamma^+ \psi(0)$, is given by

$$\langle p\lambda' | e_q J_q^+(0) | p\lambda \rangle = e_q u(p, \lambda') \gamma^+ u(p, \lambda) F^q = 2P^+ \delta_{\lambda'\lambda} e_q F^q. \quad (23)$$

The factor of 2 on the *r.h.s* of Eq. (23) comes from the relativistic normalization of states and F^q is the contribution to the proton charge from the particular quark flavor. In terms of the parton-nucleon amplitudes defined in Eq. (4), the vector current matrix element is given by

$$\begin{aligned} \langle p\lambda' | e_q J_q^+(0) | p\lambda \rangle &= -e_q \int \frac{d^4 k}{i(2\pi)^4} \text{Tr}[\gamma^+ A] = -e_q \int \frac{d^4 k}{i(2\pi)^4} \text{Tr}[\gamma^+ A^+ \frac{\not{k}}{4}] \\ &= e_q \delta_{\lambda'\lambda} \int \frac{d^4 k dm^2}{i} k^+ \left[\frac{\rho_s^+(m^2)}{(p-k)^2 - m^2 + i\epsilon} - \frac{\rho_s^+(m^2)}{-m^2 + i\epsilon} - (s \rightarrow u, k \rightarrow -k) \right] I_n \frac{1}{(k^2 - \mu^2 + i\epsilon)^2} \\ &= 2P^+ \delta_{\lambda'\lambda} e_q \int_0^1 dx [f_q(x) - \bar{f}_q(x)]. \end{aligned} \quad (24)$$

Thus as expected the quark and antiquark structure functions contribute with opposite signs. We also note that the subtraction terms do not contribute, since for these terms the integrand is antisymmetric in k^+ . The normalization of the spin dependent structure functions is related to the axial current matrix element $J_5^+(0) = \bar{\psi}(0) \gamma^+ \gamma_5 \psi(0)$

$$\langle p\lambda' | J_{5q}^+(0) | p\lambda \rangle = \bar{u}(p, \lambda') \gamma^+ \gamma_5 u(p, \lambda) g_A^q = 2P^+ g_A^q \tau_{\lambda'\lambda}^3. \quad (25)$$

In Eq. (25) g_A^q denotes the contribution from a single quark flavor to the nucleon axial charge, and in terms of

the spin-dependent structure functions should be given by

$$g_A^q = \int_0^1 dx [\Delta f_q(x) + \Delta \bar{f}_q(x)]. \quad (26)$$

Indeed, expressing the axial current matrix element in terms of the parton-nucleon amplitude we obtain

$$\begin{aligned} \langle p\lambda' | J_{5q}^+(0) | p\lambda \rangle &= - \int \frac{d^4 k}{i(2\pi)^4} \text{Tr}[\gamma^+ \gamma_5 A] = - \int \frac{d^4 k}{i(2\pi)^4} \text{Tr}[\gamma^+ A^- \frac{\not{k}}{4}] \\ &= \tau_{\lambda'\lambda}^3 \int \frac{d^4 k dm^2}{i} k^+ \left[\frac{\rho_s^-(m^2)}{(p-k)^2 - m^2 + i\epsilon} - \frac{\rho_s^-(m^2)}{-m^2 + i\epsilon} - (s \rightarrow u, k \rightarrow -k) \right] I_n \frac{1}{(k^2 - \mu^2 + i\epsilon)^2} \\ &= 2P^+ \tau_{\lambda'\lambda}^3 \int_0^1 dx [\Delta f_q(x) + \Delta \bar{f}_q(x)]. \end{aligned} \quad (27)$$

In the following Section we will consider the collinear approximation for the DVCS amplitude.

C. The DVCS Reaction $\gamma^* p \rightarrow \gamma p$

When $\Delta \neq 0$ it is convenient to choose a frame with the following momentum coordinates [36] (where again we ignore the nucleon mass), $p = [P^+, 0, 0_\perp]$, $p' = [(1 - \zeta)P^+, \Delta_\perp^2/(1 - \zeta)P^+, \Delta_\perp]$, $q = [0, (Q_\perp - \Delta_\perp)^2/\zeta P^+ + \Delta_\perp^2/(1 - \zeta)P^+, Q_\perp]$, $q' = [\zeta P^+, (Q_\perp - \Delta_\perp)/\zeta P^+, Q_\perp - \Delta_\perp]$. In the Bjorken limit, at small momentum transfer, $\zeta = x_B + O(-t/Q^2)$ and $-t = \Delta_\perp^2/(1 - \zeta)$. Since we are interested in the small- t region we also set $\Delta_\perp = 0$ which also implies $\Delta^2 = 0$ ($\Delta \rightarrow [-\zeta P^+, 0, 0_\perp]$). To facilitate comparison with standard formulas it is convenient to shift the integration variable in Eqs. (11), (12) from k to $\tilde{k} \equiv k + \Delta/2$. In the collinear approximation, the hard

propagators become

$$\begin{aligned} & \frac{1}{\left(\frac{q+q'}{2} + \tilde{k} - \Delta/2\right)^2} \pm \frac{1}{\left(\frac{q+q'}{2} - \tilde{k} + \Delta/2\right)^2} \\ &= \frac{1}{(q' + \tilde{k})^2 + i\varepsilon} \pm \frac{1}{(q - \tilde{k})^2 + i\varepsilon} \\ &= \frac{x_B}{Q^2} \left[\frac{1}{\frac{\tilde{k}^+}{P^+} + i\epsilon} \pm \frac{1}{-x_B - \frac{\tilde{k}^+}{P^+} + i\epsilon} \right]. \end{aligned} \quad (28)$$

Next we combine the two soft propagators,

$$\frac{1}{[(\Delta - \tilde{k})^2 - \mu^2] [\tilde{k}^2 - \mu^2]} = \int_0^1 dr \frac{1}{[(\tilde{k} - r\Delta)^2 - \mu^2 + i\epsilon]^2} \quad (29)$$

and for $T_+^{\mu\nu}$ we obtain,

$$\begin{aligned} T_+^{\mu\nu} &= -\delta_{\lambda'\lambda} e_q^2 \frac{1}{2} [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \int \frac{d^4 \tilde{k} dm^2}{i} \int_0^1 dr \int_0^1 dx \left[\frac{1}{\frac{\tilde{k}^+}{P^+} + i\epsilon} - \frac{1}{-x_B - \frac{\tilde{k}^+}{P^+} + i\epsilon} \right] \left(\frac{\tilde{k}^+}{P^+} - \frac{\Delta^+}{2P^+} \right) \\ &\times \left[\rho_s^+(m^2) I_n \left(\frac{2(1-x)}{[(\tilde{k} - xp' - (1-x)r\Delta]^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} - \frac{1}{-m^2[(\tilde{k} - r\Delta)^2 - \mu^2 + i\epsilon]^2} \right) \right. \\ &\left. + \rho_u^+(m^2) I_n \left(\frac{2(1-x)}{[(\tilde{k} + xp - (1-x)r\Delta]^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} - \frac{1}{-m^2[(\tilde{k} - r\Delta)^2 - \mu^2 + i\epsilon]^2} \right) \right] \end{aligned} \quad (30)$$

and after integrating over \tilde{k}^- and \tilde{k}_\perp , we obtain a formal relation that is reminiscent of the standard leading-twist DVCS formula in terms of GPD's. The hadronic tensors, spectral functions and generalized parton distributions here all represent the contribution from a single quark

flavor; we have not included the quark flavor indices but they are implicit. As will be discussed shortly, this expression for DVCS fails to be convergent in the presence of Regge behavior.

$$T_+^{\mu\nu} = -e_q^2 \delta_{\lambda'\lambda} [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \left[\int_0^1 dx H^+(x, x_B) \left(\frac{1}{x - i\epsilon} + \frac{1}{x - x_B + i\epsilon} \right) + \frac{f_0(x) + \bar{f}_0(x)}{x} \int_0^1 dr \frac{(1-2r)^2}{2r(1-r)} \right]. \quad (31)$$

Here f_0 and \bar{f}_0 are given by Eq. (18) without $(1-x)^2$ in the numerator. Just as in the symmetric case analyzed above in the context of DIS, the contribution given by the quark (antiquark) distribution $f_q(x)$ ($\bar{f}_q(x)$) comes from the s -channel (u -channel) spectral function respectively. The δ -function which arises after \tilde{k}^- integration fixes \tilde{k}^+/P^+ in terms of the Feynman parameter- x , and leads to both positive and negative \tilde{k}^+/P^+ .

We immediately note that the last term in Eq. (31), which originates from the subtractions in the parton-nucleon amplitude needed for the Regge term, not only contributes but is in fact singular, since the integral diverges for both $r \rightarrow 0$ and $r \rightarrow 1$ and it has the same sign at both limits. The generalized parton distribution

H^+ appearing in Eq. (31) is given by

$$H^+(x, x_B) = (x - x_B/2) \int_0^1 dr \int_0^1 \frac{dy}{y} \times \delta[x - y - (1 - y)rx_B] [f_q(y) + \bar{f}_q(y)] \quad (32)$$

and it is the C -even generalized parton distribution [5]. It can easily be checked that H^+ satisfies the correct normalization conditions

$$\int_0^1 dx \frac{H^+(x, x_B)}{1 - x_B/2} = \int_0^1 dx [f_q(x) + \bar{f}_q(x)] \quad (33)$$

and

$$H^+(x, 0) = f_q(x) + \bar{f}_q(x). \quad (34)$$

Even though the integrals over x and $H^+(x, 0)$ are finite, H^+ is defined by Eq. (32) which is singular. This can be seen by doing the y integral using the δ function and then changing variables to $z = (x - rx_B)/(1 - rx_B)$ and expressing Eq. (32) in the form

$$H^+(x, x_B) = \frac{x - x_B/2}{x_B} \left[\theta(x_B - x) \int_0^x \frac{dz}{z(1 - z)} [f_q(z) + \bar{f}_q(z)] + \theta(x - x_B) \int_{\frac{x - x_B}{1 - x_B}}^x \frac{dz}{z(1 - z)} [f_q(z) + \bar{f}_q(z)] \right]. \quad (35)$$

First, there is the singularity of H^+ which is of the same type as in the Regge subtraction term discussed above. It comes from the lower limit of the integral in the term proportional to $\theta(x_B - x)$ in Eq. (35). After integrating over the first hard propagator ($1/[x - i\epsilon]$) in Eq. (31), the contribution from this singularity to the hadronic tensor $T_+^{\mu\nu}$ in the region $x \sim 0$ is given by

$$-\frac{1}{2} \int_0^x \frac{dx}{x} \int_0^x \frac{dz}{z} [f_q(z) + \bar{f}_q(z)], \quad (36)$$

and after integrating over the second hard propagator $1/[x - x_B + i\epsilon]$, in the vicinity $x \sim x_B^-$ gives,

$$\begin{aligned} & +\frac{1}{2} \int^{x_B} \frac{dx}{x - x_B} \int_0^x \frac{dz}{z} [f_q(z) + \bar{f}_q(z)] = \\ & = -\frac{1}{2} \int^{x_B} \frac{dx}{x_B - x} \int_0^x \frac{dz}{z} [f_q(z) + \bar{f}_q(z)] \end{aligned} \quad (37)$$

The sum of these two exactly cancels the singularities from the Regge subtraction term.

There are however, residual singularities in the DVCS amplitude which originate from the Regge behavior of H^+ . Consider the contribution to the x -dependence of H^+ from the upper region of integration of the term proportional to $\theta(x_B - x)$ in Eq. (35). The low- x Regge behavior of the quark and antiquark structure functions is $f_q(x) \sim 1/x^{\alpha_s^+}$ and $\bar{f}_q(x) \sim 1/x^{\alpha_u^+}$, so the quark and antiquark distributions give contributions to H^+ of the general form

$$H^+(x \sim 0) \sim \frac{1}{2\alpha} \frac{1}{x^\alpha}. \quad (38)$$

The integral over the first hard propagator in Eq. (31) thus gives a contribution to the DVCS amplitude

$$\int_0^1 dx H^+(x, x_B) \frac{1}{x - i\epsilon} \sim \mathcal{O}\left(\frac{1}{\epsilon^\alpha}\right) \quad (39)$$

which is divergent for $0 < \alpha < 1$.

Similarly, as $x \rightarrow x_B^+$ the term in Eq. (35) for H^+ proportional to $\theta(x - x_B)$, by virtue of the Regge form for $f_q(z)$ and/or $\bar{f}_q(z) \propto 1/z^\alpha$, is dominated by the lower limit of the integral over z , leading to

$$H^+(x \sim x_B^+) \sim \frac{1}{2\alpha} \frac{(1 - x_B)^\alpha}{(x - x_B)^\alpha} \quad (40)$$

Using the form of H^+ from Eq. (40) in Eq. (31), and integrating over the second hard propagator, ($1/[x - x_B + i\epsilon]$), gives a contribution to the DVCS amplitude of the form

$$\int_{x_B} dx \frac{H^+(x, x_B)}{x - x_B + i\epsilon} \sim (1 - x_B)^\alpha \mathcal{O}\left(\frac{1}{\epsilon^\alpha}\right). \quad (41)$$

These residual singularities must originate from the collinear approximation since after Regge subtraction there is no reason to expect that the expression for $T_+^{\mu\nu}$ in Eq. (11) will be singular. In other words, to properly regularize those singularities it will be necessary to retain the full momentum dependence of the hard propagators.

We note that the problem arises from the Regge contribution to the soft part of the handbag diagram. The valence spectral functions do not require subtraction, thus their contributions to $T_+^{\mu\nu}$ do not have the singularity associated with the $(f_0(x) + \bar{f}_0(x))/x$ term in Eq. (31). Furthermore valence structure functions vanish at small- x . As a result, the valence contributions vanish in the regions $H^+(x \sim 0)$ and $H^+(x \sim x_B^+)$, so no singularities appear of the type given in Eqs. (39) and (41).

A similar analysis of the antisymmetric contribution yields,

$$\begin{aligned}
T_-^{\mu\nu} = & -ie_q^2 \epsilon^{\mu\nu}_{03} \tau_{\lambda'\lambda}^3 \frac{1}{2} \int \frac{d^4 \tilde{k} dm^2}{i} \int_0^1 dr \int_0^1 dx \left[\frac{1}{\frac{\tilde{k}^+}{P^+} + i\epsilon} + \frac{1}{-x_B - \frac{\tilde{k}^+}{P^+} + i\epsilon} \right] \left(\frac{\tilde{k}^+}{P^+} - \frac{\Delta^+}{2P^+} \right) \\
& \times \left[\rho_s^+(m^2) I_n \left(\frac{2(1-x)}{[(\tilde{k} - xp' - (1-x)r\Delta)^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} - \frac{1}{-m^2[(\tilde{k} - r\Delta)^2 - \mu^2 + i\epsilon]^2} \right) \right. \\
& \left. - \rho_u^+(m^2) I_n \left(\frac{2(1-x)}{[(\tilde{k} + xp - (1-x)r\Delta)^2 - xm^2 - (1-x)\mu^2 + i\epsilon]^3} - \frac{1}{-m^2[(\tilde{k} - r\Delta)^2 - \mu^2 + i\epsilon]^2} \right) \right], \quad (42)
\end{aligned}$$

$$T_-^{\mu\nu} = ie_q^2 \epsilon^{\mu\nu}_{03} \tau_{\lambda'\lambda}^3 \left[\int_0^1 dx \tilde{H}^+(x, x_B) \left(\frac{1}{x - i\epsilon} - \frac{1}{x - x_B + i\epsilon} \right) - \frac{\Delta f_0(x) - \Delta \bar{f}_0(x)}{x} \int_0^1 dr \frac{(1-2r)}{2r(1-r)} \right]. \quad (43)$$

In this case the infinities arising from the $r = 0$ and $r = 1$ points in the last term in Eq. (43) cancel each other, and the contribution from the Regge subtraction terms vanishes altogether. The \tilde{H}^+ parton distribution

appearing in Eq. (43) is given by the same equation as H^+ from Eq. (35) with the replacements $f_q \rightarrow \Delta f_q$ and $\bar{f}_q \rightarrow \Delta \bar{f}_q$,

$$\tilde{H}^+(x, x_B) = \frac{x - x_B/2}{x_B} \left[\theta(x_B - x) \int_0^x \frac{dz}{z(1-z)} [\Delta f_q(z) + \Delta \bar{f}_q(z)] + \theta(x - x_B) \int_{\frac{x-x_B}{1-x_B}}^x \frac{dz}{z(1-z)} [\Delta f_q(z) + \Delta \bar{f}_q(z)] \right] \quad (44)$$

In Eq. (44) the Regge parts of $\Delta f_q(z)$ and $\Delta \bar{f}_q(z)$ behave at small z like $1/z^{\alpha_s^-}$ and $1/z^{\alpha_u^-}$, respectively. The singularity of the first integral over z (for $x < x_B$) does not contribute to the DVCS amplitude. This is because after multiplying by the sum of the two hard propagators the contribution of this singularity to $T_-^{\mu\nu}$ in Eq. (43) becomes proportional to

$$\int_0^{x_B} \left(x - \frac{x_B}{2} \right) \left(\frac{1}{x - i\epsilon} - \frac{1}{x - x_B + i\epsilon} \right) dx = 0. \quad (45)$$

There are nevertheless the same left-over singularities in the DVCS amplitude as in the case of $T_+^{\mu\nu}$. These originate from the behavior of \tilde{H}^+ near $x \sim 0$ (from the upper limit of the integral in the $\theta(x_B - x)$ term), and from the region $x \sim x_B^+$ (from the lower limit of the integral in the term proportional to $\theta(x - x_B)$). In the region $x \sim 0$ one has the generic behavior $\tilde{H}^+ \sim 1/x^\alpha$, so the integral over the first hard propagator is of the form,

$$\int_0^{x_B} \frac{dx}{x^\alpha} \frac{1}{x - i\epsilon} = \mathcal{O} \left(\frac{1}{\epsilon^\alpha} \right). \quad (46)$$

In the region $x \sim x_B$, $\tilde{H}^+(x \sim x_B^+) \sim (1 - x_B)^\alpha / (x - x_B)^\alpha$ and the integral over the second hard propagator becomes

$$(1 - x_B)^\alpha \int_{x_B}^1 \frac{dx}{(x - x_B)^\alpha} \frac{1}{x - x_B + i\epsilon} \sim (1 - x_B)^\alpha \mathcal{O} \left(\frac{1}{\epsilon^\alpha} \right). \quad (47)$$

Even though these singular terms contribute to $T_-^{\mu\nu}$ with opposite signs they do not cancel because of the extra factor $(1 - x_B)^\alpha$.

III. DVCS AMPLITUDE WITHOUT COLLINEAR APPROXIMATION

In the previous section we noticed that the C - even part of the DVCS amplitude is singular when evaluated in collinear approximation and expressed in terms of the H^+ or \tilde{H}^+ GPD's, provided that the parton-nucleon amplitude has a high energy behavior typical of hadronic amplitudes, commensurate with the Regge type scaling behavior of the form s^α with $0 < \alpha < 1$ (we also showed

that this problem does not arise for the structure functions). From the discussion above it is also clear that the singularity in the DVCS amplitude has to do with the collinear approximation to the denominators of the hard quark propagator exchanged between photon interaction points. Thus in the following we use the collinear

approximation for the numerators and keep the full k -dependence of the denominators while performing the d^4k integral in Eqs. (11) and (12). Then the Regge part of the spectral function, that is now finite and dominant at low t in the DVCS amplitude $T_+^{\mu\nu}$ gives,

$$\begin{aligned}
T_+^{\mu\nu} = & -\delta_{\lambda'\lambda} e_q^2 \frac{1}{2} [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \frac{Q^2}{x_B} \int_0^\infty d\xi \int_0^1 \frac{dx}{x} \int_0^1 dr \int_0^1 dz 2\pi^2 (1-x)(1-z)^2 \\
& \left\{ \frac{\mu^2 \beta_s^+}{(x\mu^2)^{\alpha_s^+}} I_{n-1} \left[\left(\frac{\xi^{\alpha_s^+} (1-x)^2 [x(1-x_B) + \frac{x_B}{2} - (1-x)(1-z)rx_B - (1-x)zx_B]}{[\xi + (1-x)(1-z)\mu^2 - (2p \cdot q + q^2)xz(1-x) - z(1-z)(1-x)^2rq^2 - i\epsilon]^3} - (x=0) \right) \right. \right. \\
& - \left. \left(\frac{\xi^{\alpha_s^+} (1-x)^2 (x(1-x_B) + \frac{x_B}{2} - (1-x)(1-z)rx_B)}{[\xi + (1-x)(1-z)\mu^2 + (2p \cdot q)xz(1-x) - z(1-z)(1-x)^2(1-r)q^2 - i\epsilon]^3} - (x=0) \right) \right] \\
& + \frac{\mu^2 \beta_u^+}{(x\mu^2)^{\alpha_u^+}} I_{n-1} \left[\left(\frac{\xi^{\alpha_u^+} (1-x)^2 (-x + \frac{x_B}{2} - (1-x)(1-z)rx_B - (1-x)zx_B)}{[\xi + (1-x)(1-z)\mu^2 + (2p \cdot q)xz(1-x) - z(1-z)(1-x)^2rq^2 - i\epsilon]^3} - (x=0) \right) \right. \\
& - \left. \left(\frac{\xi^{\alpha_u^+} (1-x)^2 (-x + \frac{x_B}{2} - (1-x)(1-z)rx_B)}{[\xi + (1-x)(1-z)\mu^2 - (2p \cdot q + q^2)xz(1-x) - z(1-z)(1-x)^2(1-r)q^2 - i\epsilon]^3} - (x=0) \right) \right] \Bigg\}. \tag{48}
\end{aligned}$$

Here following Ref. [29] we changed the m^2 variable to ξ , with $m^2 \rightarrow \xi/x$. The large m^2 contribution to the integral corresponds to small- x thus we ignore x in all terms of the form $(1-x)$, and terms proportional to x in the numerator, and we extend the x integral to infinity. We change the x variable so as to bring each denominator to the same form as in the subtraction terms (those with $(x=0)$). In particular for the term written explicitly in the second line of Eq. (48) we replace $x \rightarrow x'$ given by,

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)rq^2}{z(2p \cdot q + q^2)}, \tag{49}$$

in the third line,

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)(1-r)q^2}{2p \cdot qz}, \tag{50}$$

in the fourth line

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)rq^2}{2p \cdot qz}, \tag{51}$$

and in the fifth line

$$x = x' \frac{\xi + (1-z)\mu^2 - z(1-z)(1-r)q^2}{z(2p \cdot q + q^2)}. \tag{52}$$

We note that since $q^2 < 0$ and $2p \cdot q + q^2 > 0$ these transformations are non-singular. After this change of variables we obtain,

$$\begin{aligned}
T_+^{\mu\nu} = & -\delta_{\lambda'\lambda} e_q^2 \frac{1}{2} [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] Q^2 \int_0^\infty d\xi \int_0^\infty \frac{dx'}{x'} \int_0^1 dr \int_0^1 dz 2\pi^2 (1-z)^2 \\
& \left\{ \frac{\mu^2 \beta_s^+}{(x' \mu^2)^{\alpha_s^+}} I_{n-1} \left[[(2p \cdot q + q^2)z]^{\alpha_s^+} \frac{\xi^{\alpha_s^+} [\frac{1}{2} - (1-z)r - z]}{[\xi + (1-z)\mu^2 + z(1-z)rQ^2]^{3+\alpha_s^+}} \left(\frac{1}{(1-x'-i\epsilon)^3} - 1 \right) \right. \right. \\
& - [(2p \cdot q)z]^{\alpha_s^+} \frac{\xi^{\alpha_s^+} [\frac{1}{2} - (1-z)r]}{[\xi + (1-z)\mu^2 + z(1-z)(1-r)Q^2]^{3+\alpha_s^+}} \left. \left(\frac{1}{(1+x'-i\epsilon)^3} - 1 \right) \right] \\
& + \frac{\mu^2 \beta_u^+}{(x' \mu^2)^{\alpha_u^+}} I_{n-1} \left[[(2p \cdot q)z]^{\alpha_u^+} \frac{\xi^{\alpha_u^+} (\frac{1}{2} - (1-z)r - z)}{[\xi + (1-z)\mu^2 + z(1-z)rQ^2]^{3+\alpha_u^+}} \left(\frac{1}{(1+x'-i\epsilon)^3} - 1 \right) \right. \\
& \left. \left. - [(2p \cdot q + q^2)z]^{\alpha_u^+} \frac{\xi^{\alpha_u^+} (\frac{1}{2} - (1-z)r)}{[\xi + (1-z)\mu^2 + z(1-z)(1-r)Q^2]^{3+\alpha_u^+}} \left(\frac{1}{(1-x'-i\epsilon)^3} - 1 \right) \right] \right\}
\end{aligned} \tag{53}$$

The ξ integral in Eq. (53) can be performed analytically yielding,

$$\begin{aligned}
T_+^{\mu\nu} = & -\delta_{\lambda'\lambda} e_q^2 \frac{1}{2} [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] Q^2 \int_0^\infty dx' \int_0^1 dr \int_0^1 dz 2\pi^2 \\
& \left\{ \frac{z^{\alpha_s^+} \beta_s^+ (\mu^2)^{1-\alpha_s^+}}{(1+\alpha_s^+)(2+\alpha_s^+)} I_{n-1} \frac{\frac{1}{2} - (1-z)r - z}{[\mu^2 + zrQ^2]^2} \frac{1}{x'^{1+\alpha_s^+}} \left[[2p \cdot q + q^2]^{\alpha_s^+} \left(\frac{1}{(1-x'-i\epsilon)^3} - 1 \right) + [2p \cdot q]^{\alpha_s^+} \left(\frac{1}{(1+x'-i\epsilon)^3} - 1 \right) \right] \right. \\
& \left. + \frac{z^{\alpha_u^+} \beta_u^+ (\mu^2)^{1-\alpha_u^+}}{(1+\alpha_u^+)(2+\alpha_u^+)} I_{n-1} \frac{\frac{1}{2} - (1-z)r - z}{[\mu^2 + zrQ^2]^2} \frac{1}{x'^{1+\alpha_u^+}} \left[[2p \cdot q]^{\alpha_u^+} \left(\frac{1}{(1+x'-i\epsilon)^3} - 1 \right) + [2p \cdot q + q^2]^{\alpha_u^+} \left(\frac{1}{(1-x'-i\epsilon)^3} - 1 \right) \right] \right\}
\end{aligned} \tag{54}$$

The crucial ingredient which leads to the difference between the DVCS amplitude given in Eq. (54) and the DIS case studied in [29] is the presence of the r factor in the zrQ^2 terms in the denominators. In the absence of this factor, the integral over z would be dominated by the region $z \sim \mu^2/Q^2$. In that case the factors of $z^\alpha (2p \cdot q + q^2)^\alpha$ and $z^\alpha (2p \cdot q)^\alpha$ would become Q^2 independent; this would produce a Q^2 -independent expression for $T_+^{\mu\nu}$ as expected

from scaling. This additional r -dependence is of the same type as found in Ref. [24]. In that paper, Regge behavior was introduced by utilizing a t -channel approach, and not through the s or u -channel formalism as employed here. The factor of r in zrQ^2 makes r peak around μ^2/Q^2 , and this produces an overall $(Q^2)^\alpha$ dependence for the DVCS amplitude. In particular we can write the symmetric tensor $T_+^{\mu\nu}$ in the form

$$T_+^{\mu\nu} = -\delta_{\lambda'\lambda} e_q^2 [n^\mu \tilde{p}^\nu + n^\nu \tilde{p}^\mu - g^{\mu\nu} (n \cdot \tilde{p})] \left[\left(\frac{Q^2}{x_B \mu^2} \right)^{\alpha_s^+} F_s^+(x_B) + \left(\frac{Q^2}{x_B \mu^2} \right)^{\alpha_u^+} F_u^+(x_B) \right] \tag{55}$$

where in Eq. (55) we have introduced the quantities

$$F_{s,u}^+(x_B) \equiv \frac{\pi^2}{2} \frac{(1 - \alpha_{s,u}^+) \Gamma(\alpha_{s,u}^+)}{(1 + \alpha_{s,u}^+) \Gamma(3 + \alpha_{s,u}^+)} \beta_{s,u}^+ \left[\mu^2 I_{n-1} \frac{1}{\mu^2} \right] \left[\xi_{\alpha_{s,u}^+}^+ + (1 - x_B)^{\alpha_{s,u}^+} \xi_{\alpha_{s,u}^+}^- \right] \quad (56)$$

and in Eq. (56) we define

$$\xi_{\alpha}^{\pm} \equiv \int_0^{\infty} \frac{dx'}{x'^{1+\alpha}} \left[\frac{1}{(1 \pm x' - i\epsilon)^3} - 1 \right]. \quad (57)$$

We call the functions $F(x_B)$ introduced in Eq. (56) the "Regge Exclusive Amplitudes," since they contain the information from the coupling of the relevant Regge trajectories to a particular exclusive process, in this case DVCS. Unfortunately, the loss of factorization in this process makes this and analogous functions non-universal, unlike the generalized parton distributions or

GPDs. However the Regge Exclusive Amplitudes do convey information regarding the exponents α of the relevant Regge trajectories that are indeed universal. These amplitudes also allow a comparison between hard exclusive processes and high-energy total cross-sections. Alternatively one can directly employ the t -channel formulation of the hard process in terms of a single (or a few) Regge trajectories.

Finally a similar analysis for the antisymmetric amplitude yields a form

$$T_-^{\mu\nu} = ie_q^2 \epsilon^{\mu\nu}_{03} \tau_{\lambda'\lambda}^3 \left[\left(\frac{Q^2}{x_B \mu^2} \right)^{\alpha_s^-} F_s^-(x_B) + \left(\frac{Q^2}{x_B \mu^2} \right)^{\alpha_u^-} F_u^-(x_B) \right] \quad (58)$$

where the relevant Regge Exclusive Amplitudes are defined as

$$F_{s,u}^-(x_B) \equiv \frac{\pi^2}{2} \frac{(1 - \alpha_{s,u}^-) \Gamma(\alpha_{s,u}^-)}{(1 + \alpha_{s,u}^-) \Gamma(3 + \alpha_{s,u}^-)} \beta_{s,u}^- \left[\mu^2 I_{n-1} \frac{1}{\mu^2} \right] \left[\xi_{\alpha_{s,u}^-}^+ - (1 - x_B)^{\alpha_{s,u}^-} \xi_{\alpha_{s,u}^-}^- \right]. \quad (59)$$

We note the familiar structure. The finite constants ξ_{α}^{\pm} encode the integration over the hard propagators from the collinear approximation, and contribute with a relative factor of $\pm(1 - x_B)^{\alpha}$ to the symmetric and antisymmetric DVCS amplitudes, respectively. This is the same factor that arises from the singularities of the collinear approximation. The regularization of the collinear approximation leads to an increase of the hard exclusive amplitude by a factor of $(Q^2/x_B \mu^2)^{\alpha}$ relative to the DIS amplitude. This is the same enhancement factor that was found in Ref. [24]. In the more general case, when the nucleon and/or quark masses are kept finite or more than one scale appears in the parton-nucleon amplitude, the single quantity μ would be replaced by some combination of quantities. The functions $F_{s,(u)}^+$ describe the quark (antiquark) helicity averaged contribution to the DVCS amplitude. Similarly, the functions $F_{s,(u)}^-$ describe the quark (antiquark) helicity-dependent contribution to

the DVCS amplitude.

As was discussed in Section I, we have carried out a preliminary study of photon-induced exclusive processes. We have shown that Regge amplitudes should make significant contributions at large values of x_B , and not just at small x_B . A major result of our formalism is the prediction of scaling violation in these hard exclusive processes. At intermediate energies the Bethe-Heitler (BH) amplitude is generally substantially larger than DVCS, so DVCS amplitudes must be extracted via their interference with the BH term. A group at Hall A in Jefferson Laboratory [16] has recently performed a test of QCD scaling in spin-dependent $\bar{e}p$ scattering. They measured the beam-spin azimuthal asymmetry [4, 5], which is proportional to interference between BH and DVCS amplitudes. After removing the Q^2 -dependence associated with the BH term, they extracted twist-2 and twist-3 Compton form factors which by QCD scaling should be Q^2 independent. In Fig. 3 we plot the twist-2 Comp-

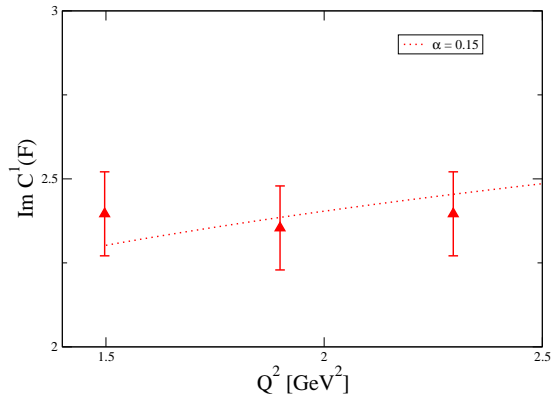


FIG. 3: (color online) Comparison with DVCS results from Jefferson Lab [16]. The data points represent the twist-2 Compton form factor extracted from beam-spin asymmetry measurements in $\bar{e}p$ scattering, vs. Q^2 . The data have been averaged over t . The dotted curve is a $(Q^2)^\alpha$ fit with $\alpha = 0.15$.

ton form factor $\mathcal{C}^I(\mathcal{F})$ vs. Q^2 ; this term has been averaged over t . Although the data show very little Q^2 dependence, they correspond to a limited range of Q^2 and are also in good agreement with our predicted behavior $(Q^2)^\alpha$. In Fig. 3 the dotted line corresponds to $(Q^2)^\alpha$ with $\alpha = 0.15$. Because the data points were averaged over t it is not obvious what value of α to choose, but over this range of Q^2 our predicted behavior is in agreement with the Hall A points.

In Fig. 4 we compare our predictions with the data on exclusive meson electroproduction. Scaling arguments predict that the reduced π^+ cross section should fall off at fixed x_B as $1/Q^2$. We predict a behavior $(Q^2)^{2\alpha-1}$ with $0 < \alpha < 1$. Fitting π^+ data from HERMES [14] in the range $0.26 < x_B < 0.8$ gives $\alpha = 0.13 \pm 0.1$. Similarly for ω electroproduction cross section from the CLAS collaboration at Jefferson Lab [13] we find $\alpha = 0.6 \pm 0.4$ for the range $0.52 < x_B < 0.58$.

We see that for both DVCS and exclusive meson electroproduction, not only are the data consistent with scaling violations, but the additional Q^2 dependence is softer than predicted by scaling and in agreement with our predicted factor of $(Q^2)^\alpha$ with $0 < \alpha < 1$. At this point it is difficult to compare the Regge exponents α obtained from the fit with total cross-section data, since the electroproduction data was taken at different values of t . However we find this trend encouraging, and we believe that it warrants further phenomenological studies. QCD scaling predicts that agreement with scaling should become progressively better with increasing Q^2 . However we have shown that scaling violations should persist regardless of the size of Q^2 .

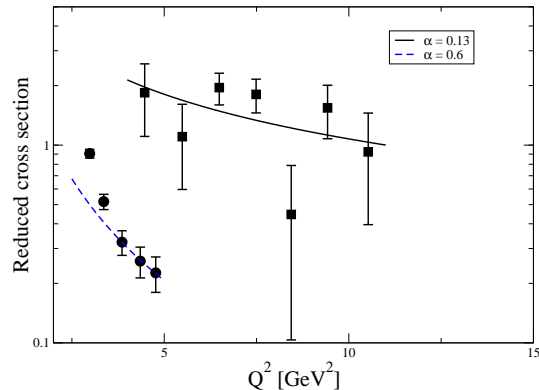


FIG. 4: (color online) A simple fit to electroproduction data for mesons, π^+ results from HERMES (squares, [14]), and ω (circles, [13]) results from the CLAS Collaboration at Jefferson Lab. In the case of π^+ production the cross section reduced by the photon flux is plotted (in arbitrary units).

IV. SUMMARY AND OUTLOOK

Our formalism in this paper started with the generic hadronic tensor for DVCS reactions. This was then expressed in terms of a parton-proton Green's function whose Regge behavior has been examined in the literature. We have shown how this makes the standard factorization formula ill-defined due to a collinear divergence brought about by this Regge behavior; this was demonstrated in Eq. (31). Appropriate regularization of this divergence, as given in Eq. (55), shows that the assumption of x_B -scaling of DVCS breaks down. Note that the general analysis of these reactions in terms of GPDs assumes x_B -scaling. In our analysis, DVCS and similar hard exclusive processes are then characterized by a new set of process-dependent Regge Exclusive Amplitudes $F(x_B)$ that are derived in Eqs. (56) and (59). Unlike GPD's, these Regge Exclusive Amplitudes are non-universal. One experimental signature of this approach is our demonstration that the Q^2 dependence of hard exclusive amplitudes should differ from scaling predictions, and that one should observe a behavior $(Q^2)^\alpha \propto s_{\gamma^*p}^{\alpha}$ characteristic of hadronic Regge amplitudes. A preliminary examination of experimental data on DVCS and hard meson production suggests that the data is consistent with the Regge non-scaling $(Q^2)^\alpha$ behavior predicted here.

We argue that the QCD factorization theorems for exclusive processes [8, 9] should not be applicable to hard exclusive processes, at least not in the region of small t . This is due to the lack of convergence of the residues of the poles in the k^- plane that appear in their derivations. This is a generic feature due to Regge behavior. Thus it appears that the QCD factorization theorems for photon-proton exclusive processes necessitate, in ad-

dition to a hard scale Q^2 , a sizeable momentum transfer t . This occurs because the Regge nature of hadron-hadron amplitudes [37] forces a corresponding behavior on the parton-nucleon scattering amplitude, which generates divergences in the Generalized Parton Distribution functions at low t . As t (or t_{min}) increases the intercept of Regge trajectories become negative, and standard collinear factorization then becomes applicable. Thus it appears that momentum transfer will become a crucial parameter in these reactions. At small t we predict sizeable effects due to scattering off the meson-cloud of the proton, while at large t the dominant effect will become scattering from the quarks in the "bare" nucleon. The extension of our results to large t and the detailed interplay with the $J = 0$ fixed pole will be examined in a future publication [38].

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